Exercises for Stochastic Processes

Tutorial exercises:

T1. Show that the normal and Poisson distributions are infinitely divisible.

T2. Determine all infinitely divisible random variables with finite support.
(Hint: Consider the Lindeberg-Feller central limit theorem.)

T3. Let N be a Poisson process with intensity $\lambda > 0$, $Y_1, Y_2, \ldots$ i.i.d. and $N, Y_1, Y_2, \ldots$ independent. Show "by hand" that the "compound Poisson process" given by

$$X_t := \sum_{n=1}^{N_t} Y_n$$

has stationary and independent increments.

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Homework exercises:

H1. (a) We consider Brownian motion with reflection at 0:

$$X_t := |B_t|,$$

where B_t is Brownian motion started at $x \geq 0$. Show that the generator for this process is $\mathcal{L}_r f = \frac{1}{2} f''$ on the domain

$$\{\mathcal{D}(\mathcal{L}_r) = \{f \in C_0[0,\infty) \ : \ f', f'' \in C_0[0,\infty), f'(0) = 0\},$$

where the first and second derivative at 0 are to be understood as right-hand derivatives.

Hint: for $f \in C[0,\infty)$, consider the even extension to the entire real line:

$$f_e(x) := \begin{cases} f(x) & \text{if } x \geq 0, \\ f(-x) & \text{if } x < 0. \end{cases}$$

(b) Consider Brownian motion with absorption at 0:

$$X_t := B_t \mathbb{1}_{\{t \leq \tau\}},$$

where τ is the first hitting time of 0, and B_t Brownian motion started at $x \geq 0$. What is the generator and corresponding domain for this process?

Hint: for $f \in C[0,\infty)$, consider the odd extension to the entire real line:

$$f_o(x) := \begin{cases} f(x) & \text{if } x \geq 0, \\ 2f(0) - f(-x) & \text{if } x < 0. \end{cases}$$

Deadline: Monday, 14.01.20